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ABSTRACT

The use of two-stage least squares (2SLS) for the estimation of feedback linkages is inappropriate for nonorthogonal data sets because 2SLS is extremely sensitive to multicollinearity. It is argued that what is needed is use of a different estimating criterion than the least squares criterion. Theoretically the variance normalization criterion has superior properties. When two-stage normalization regression (2SNR), which uses the variance normalization criterion, is used to estimate a psycholinguistic model of grade 4 literacy and numeracy, the multiple feedback parameters were shown to be more stable than those generated by the 2SLS technique. Thus, at the grade 4 level intuitive linguistic strategies were more influential in accounting for literacy and numeracy than formally learned strategies. The relationships between basic skills on the one hand and between literacy and numeracy on the other were mutually reinforcing, and the basic skills were found to be important variables mediating the relationship between cueing strategies and the acquisition of literacy and numeracy. Though the 2SNR technique in practice seemed to have important advantages over 2SLS, Monte Carlo simulations have to be conducted before it can be recommended as the standard nonrecursive estimating procedure. (Author/RL)

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THE PROBLEM OF MULTIPLE FEEDBACK ESTIMATION*

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ABSTRACT

The use of two-stage least squares (2SLS) for the estimation of feedback linkages is inappropriate for nonorthogonal data sets because 2SLS is extremely sensitive to multicollinearity. It is argued that what is needed is use of a different estimating criterion than the least squares criterion. Theoretically the variance normalization criterion has superior properties. When two-stage normalization regression (2SNR), which uses the variance normalization criterion, is used to estimate a psycholinguistic model of grade four literacy and numeracy the multiple feedback parameters were shown to be more stable than those generated by the 2SLS technique.

THIS PAPER IS ABOUT THE ESTIMATION OF FEEDBACK PARAMETERS IN SOCIAL SCIENCE MODELS.

FIRST THE PROBLEM IS IDENTIFIED.

SECOND, A SOLUTION TO THE PROBLEM IS PROPOSED.

THIRD, THE PROPOSED SOLUTION IS APPLIED TO A PRACTICAL PROBLEM IN SOCIAL MODELING.

INCREASINGLY, SOCIAL SCIENCE MODELS CALL FOR RECIPROCAL RELATIONSHIPS BETWEEN VARIABLES. CONSIDER THE COMMON SOCIAL PSYCHOLOGICAL PROBLEM OF THE RELATIONSHIP BETWEEN ATTITUDES AND BEHAVIOUR. IT IS IN ORDER FOR A BEHAVIOURAL RESPONSE TO BE CONCEIVED OF AS A STIMULI FOR A CHANGE IN ATTITUDE AND VICE VERSA. ACHIEVEMENT STIMULATES ACHIEVEMENT; THAT IS, IS MUTUALLY REINFORCING. BLALOCK GIVES THE EXAMPLE OF AN ARM'S RACE, IN WHICH THE LEVEL OF ARMS IN EACH NATION IS A FUNCTION OF THE LEVELS IN OTHER NATIONS.

I. THE PROBLEM

THOSE WHO HAVE TRIED TO ESTIMATE FEEDBACK PARAMETERS IN CAUSAL MODELS WILL KNOW FROM FIRST HAND EXPERIENCE HOW DIFFICULT IT CAN BE TO INTERPRET FEEDBACK ESTIMATES.

USUALLY A TWO-STAGE LEAST SQUARES (2SLS) ESTIMATING TECHNIQUE IS USED BY THE SOCIOLOGIST, THOUGH OTHER METHODS OF ESTIMATION INCLUDE THREE-STAGE LEAST SQUARES AND FULL INFORMATION MAXIMUM LIKELIHOOD APPROACHES.

DISCUSSION HERE WILL BE LIMITED TO TO THE 2SLS METHOD. THE PROBLEM CAN BE ACUTE REGARDLESS OF WHICH ESTIMATING PROCEDURE IS USED.

THOUGH 2SLS PERFORMS WELL IN TERMS OF BIAS AND MEAN SQUARE ERROR; AND THOUGH IT IS NOT EXCLUSIVELY SENSITIVE TO SPECIFICATION ERROR, IT IS EXTREMELY SENSITIVE TO MULTICOLLINEARITY.

MULTICOLLINEARITY REFERS TO THE PRESENCE OF HIGHLY INTERCORRELATED INDEPENDENT VARIABLES IN STRUCTURAL EQUATION MODELS. ITS EFFECT IS TO INVALIDATE SOME OF THE BASIC ASSUMPIONS ON WHICH LEAST SQUARES AND MAXIMUM LIKELIHOOD ESTIMATING METHODS ARE BASED. MULTICOLLINEARITY IS A PERVASIVE PROBLEM IN SOCIAL MODELING. IT IS NOT ONE OF KIND -- OF EXISTENCE OR NONEXISTENCE -- BUT RATHER ONE OF DEGREE.

THE HARMFUL EFFECTS OF MULTICOLLINEARITY PRODUCE ESTIMATES THAT ARE: (1) TOO LARGE IN MAGNITUDE, (2) TOO SENSITIVE TO SAMPLING ERROR, AND (3) TOO SENSITIVE TO SPECIFICATION ERROR.

SUCH SENSITIVITY RESULTS IN (1) WRONG SIGNS ON SOME COEFFICIENTS, (2) DRASTIC CHANGES IN REGRESSION COEFFICIENTS AFTER MINOR DATA REVISION, (3) CONFLICTING CONCLUSIONS FROM THE USUAL SIGNIFICANCE TESTS, AND (4) INFLATED VARIANCE OF THE ESTIMATES.

THUS, IN THE PRESENCE OF ACUTE MULTICOLLINEARITY THE REGRESSION ESTIMATES ARE BOTH ERRONEOUS AND UNINTERPRETABLE.

II. What Can Be Done?

IN ANY ESTIMATING PROCEDURE IT IS DESIRABLE TO KEEP THE VARIANCE AS SMALL AS POSSIBLE.

THE TOTAL VARIANCE OF THE REGRESSION COEFFICIENTS MAY BE EXPRESSED AS:

$$\Sigma \text{VAR}(\beta) = \sigma^2 \text{TR}(X'X)^{-1}$$

$$= \frac{\epsilon'\epsilon}{N - P} \text{TR}(X'X)^{-1} \qquad (1)$$

WHERE Σ REPRESENTS THE SUMMATION FROM $i = 1$ TO $i = P$, THE NUMBER OF PREDICTORS;
WHERE β IS THE $(K \times 1)$ VECTOR OF ESTIMATED REGRESSION COEFFICIENTS;
WHERE ϵ IS THE RANDOM ERROR OF Y IN THE STANDARDIZED REGRESSION MODEL, $Y = X\beta + \epsilon$; WHERE N = SAMPLE SIZE; AND $\text{TR}(X'X)^{-1}$

IS THE TRACE OF THE INVERSE OF THE CORRELATION MATRIX.

THUS, THE TOTAL VARIANCE OF THE ESTIMATES DEPENDS ON THREE INDEPENDENT FACTORS; NAMELY, $\epsilon'\epsilon$, N - P, AND $TR(X'X)^{-1}$.

THE STANDARD SOLUTIONS TO THE VARIANCE INFLATION CAUSED BY MULTICOLLINEARITY DEPEND ON LOCATING THESE FACTORS AND ELIMINATING THEM (IF POSSIBLE).

THE FIRST SOURCE OF INFLATED VARIANCE IS $\epsilon'\epsilon$, THE RANDOM ERROR. THIS IS THE RANDOM PORTION OF THE DEPENDENT VARIABLE, Y. IT GENERALLY CONSISTS OF TWO PARTS; THAT DUE TO MEASUREMENT ERROR ϵ_m, AND THAT DUE TO STOCHASTIC ERROR, ϵ_s. STOCHASTIC ERROR IS ALSO ATTRIBUTABLE TO TWO COMPONENTS; NAMELY, THE INCOMPLETENESS OF THE DESIGNED MODEL AND SOME UNKNOWN IRREPRODUCIBLE FLUCTUATION.

THESE TWO ERRORS ARE ASSUMED TO BE UNCORRELATED AND TO HAVE NORMAL DISTRIBUTION WITH ZERO MEAN. THUS, THE SUM OF THE SQUARE ERROR MAY BE EXPRESSED AS:

$$\epsilon'\epsilon = \epsilon_m'\epsilon_m + \epsilon_s'\epsilon_s \qquad (2)$$

THIS ERROR MAY BE REDUCED THOUGH NOT ELIMINATED BY IMPROVING THE PRECISION OF MEASUREMENT, AND BY IMPROVING THE DESIGN OF THE MODEL; THAT IS, BY IMPROVED MODEL SPECIFICATION.

THE SECOND SOURCE OF INFLATED VARIANCE IS THE SAMPLE SIZE OR, MORE SPECIFICALLY, THE DEGREE OF FREEDOM OF THE SUM OF THE SQUARES ERROR (N - P).

THUS, TOTAL VARIANCE MAY BE REDUCED BUT NOT ELIMINATED BY INCREASING SAMPLE SIZE.

THE THIRD AND MOST IMPORTANT SOURCE OF VARIANCE INFLATION IS DUE TO THE EFFECTS OF MULTICOLLINEARITY WHICH ENTERS THROUGH $TR(X'X)^{-1}$; THAT IS, THE SUM OF THE VARIANCE INFLATION FACTOR (VIF) OF EACH VARIABLE DUE TO ITS INTERRELATIONSHIP WITH THE REST OF THE EXPLANATORY VARIABLES. THIS SOURCE OF VARIANCE IS STATISTICAL, THUS A STATISTICAL SOLUTION IS CALLED FOR.

III. Statistical Solution #1: Ridge Regression (RR)

TIME DOES NOT ALLOW A DETAILED DESCRIPTION OF RIDGE REGRESSION (RR). SUFFICE IT TO SAY THAT RR INVOLVES AUGMENTING THE DIAGONAL OF THE NORMAL EQUATION MATRIX WITH A SMALL POSITIVE QUANTITY, CALLED "K" (A CONSTANT), INORDER TO PRODUCE REGRESSION COEFFICIENTS WITH SMALLER VARIANCE AT THE EXPENSE OF INTRODUCING A SMALL BIAS.

THE ARGUMENT IS THAT AN ESTIMATOR THAT HAS A SMALL BIAS AND A SMALL VARIANCE IS TO BE PREFERRED TO ONE THAT HAS NO BIAS BUT A LARGE VARIANCE.

THE ARGUMENT HAS BEEN STRONGLY SUPPORTED IN MONTE CARLO

EXPERIMENTS IN WHICH THE TRUE REGRESSION COEFFICIENTS ARE SPECIFIED. IN THESE EXPERIMENTS THE ESTIMATED REGRESSION COEFFICIENTS USING RR AND OLS ARE COMPARED IN TERMS OF MEAN SQUARED ERROR (MSE); THAT IS, THE AVERAGE SQUARED DISTANCE BETWEEN ESTIMATE AND PARAMETER. VINOD (1978) REPORTS SIXTEEN MONTE CARLO EXPERIMENTS CONDUCTED BY INVESTIGATORS FROM DIVERSE FIELDS. HE STATES, "THE SUPERIORITY OF RR OVER OLS IS ALMOST ALWAYS NOTED."

THUS:

(1) THE RIDGE ESTIMATE IS LESS SENSITIVE TO SAMPLING FLUCTUATION.

(2) RR PRODUCES SMALLER SAMPLING VARIANCE THAN OLS REGRESSION.

(3) RR IS LESS SENSITIVE TO MODEL MISSPECIFICATION.

(4) RIDGE ESTIMATES ARE MORE ACCURATE.

(5) THE RIDGE TECHNIQUE GENERATES A SHORTER COEFFICIENT VECTOR.

(6) RR ALWAYS PRODUCES A MORE ACCURATE PREDICTION EQUATION IF THE BIASING PARAMETER IS NOT TOO LARGE.

YET, RR IS NOT FOOLPROOF. THERE ARE FOUR DILEMMAS.

(1) THE THEORETICAL VALUE OF THE OPTIMUM BIASING PARAMETER "K" OF RR IS STOCHASTIC WHEN IT SHOULD BE NONSTOCHASTIC. THUS, RR DOES NOT PROVIDE A SINGLE, AUTOMATIC SOLUTION TO THE ESTIMATION PROBLEM; BUT RATHER A FAMILY OF SOLUTIONS. AT THE LAST COUNT THERE WERE FIFTEEN RR METHODS, EACH YIELDING A DIFFERENT SOLUTION. THUS, THERE IS WIDE DISAGREEMENT ABOUT THE OPTIMUM RR METHOD.

(2) THE OPTIMUM "K" DEPENDS ON UNKNOWN POPULATION PARAMETERS; HENCE, CAN ONLY BE ESTIMATED.

(3) EVEN FOR AN ORTHOGONAL DATA SET (THE BOUNDARY CONDITION) THERE IS AN OPTIMUM "K" AT WHICH MSE IS MINIMUM, THOUGH IN THEORY FOR THE BOUNDARY CASE "K" SHOULD BE EQUAL TO ZERO.

(4) THE BIAS OF THE RIDGE ESTIMATES CANNOT BE CALCULATED; THUS, THE PERFORMANCE OF RR CANNOT BE EVALUATED.

IV. STATISTICAL SOLUTION #2: NORMALIZATION RIDGE REGRESSION (NR)

CAN AN ESTIMATING PROCEDURE BE DEVELOPED WHICH RETAINS THE ADVANTAGES OF THE RR PROCEDURE WHILE AT THE SAME TIME OVERCOMING ITS LIMITATIONS?

THE FIRST THREE OF THE ABOVE DILEMMAS OF RR STEM FROM USE OF THE MSE CRITERION. THE QUESTION WHICH ARISES IS WHETHER AN ALTERNATIVE CRITERION CAN BE FOUND.

MARQUARDT AND SNEE (1975) PROPOSE A VALUE FOR THE BIASING PARAMETER "K" FOR WHICH THE MINIMUM VARIANCE INFLATION FACTOR IS "BETWEEN ONE AND TEN AND CLOSER TO ONE."

THE LOWEST PERMISSIBLE TOTAL VARIANCE IN AN EQUATION IS $P\sigma^2$. THIS IS THE VALUE OF THE BEST CONDITION; NAMELY,

$$\Sigma \text{VAR}\,(\hat{\beta}^{*}) = p\sigma^{-2}$$

BUT, $$\sigma^{2} \Sigma \frac{\lambda_i}{(\lambda_i + k)^2} = p\sigma^{-2}$$

THEREFORE, $$\frac{1}{p} \Sigma \frac{\lambda_i}{(\lambda_i + k)^2} = 1 \qquad (3)$$

IN OTHER WORDS, IN ORDER TO REDUCE VARIANCE INFLATION DUE TO MULTICOLLINEARITY ONE SHOULD NORMALIZE THE VARIANCE INFLATION FACTOR (VIF) SO THAT IT IS EQUAL TO ONE. THE VALUE OF "K" WHICH SATISFIES THIS CONDITION IS THE "K" WHICH SATISFIES EQ. (3).

THE VIF OF EACH TERM IS A MEASURE OF THE COLLECTIVE IMPACT OF THE INTERDEPENDENCY OF THE EXPLANATORY VARIABLES ON THE VARIANCE OF THE REGRESSION COEFFICIENTS OF THAT TERM. ITS VALUE GIVES AN INDICATION OF THE NUMBER OF TIMES THE VARIANCE HAS BEEN INFLATED.

THUS, WHEN THE VARIANCE NORMALIZATION CRITERION IS USED THE FIRST THREE OF THE FOUR DILEMMAS OF RR ARE CORRECTABLE.

WHAT IS THE RELEVANCE OF NORMALIZATION REGRESSION (NR) FOR FEEDBACK ESTIMATION? THE ANSWER IS THAT THE SENSITIVITY OF THE 2SLS TECHNIQUE TO MULTICOLLINEARITY IS CORRECTABLE IF THE ANALYST USES THE VARIANCE NORMALIZATION CRITERION IN PREFERENCE TO THE LEAST SQUARES CRITERION. IN SUCH AN INSTANCE THE ANALYST MAY REFER TO THE APPROACH AS TWO STAGE NORMALIZATION REGRESSION (2SNR).

V. 2SLS vs. 2SNR in Practice

Consider a model of grade four literacy and numeracy in which it is hypothesized that:

(1) The pupil's intuitive linguistic rule-following strategies (namely, the application of syntactic and semantic rules) affect the acquisition of literacy and numeracy;

(2) The pupil's formally learned linguistic strategies (namely, the application of graphic and phonic rules) also affect the acquisition of literacy and numeracy;

(3) These relationships (hypotheses one and two) are mediated by teacher treatments which take the form of promoting the basic skills (namely, vocabulary, language, and work study skills);

(4) The basic skills are reciprocal on the grounds that achievement stimulates achievement;

(5) Literacy and numeracy as indicated by performances in reading and arithmetical problem solving are also reciprocal;

(6) Achievement in reading and arithmetic are more dependent on the intuitively acquired rules than the formally acquired rules; and

(7) The cueing strategies (the application of graphic, phonic, and syntactic-semantic rules) and basic skills will account for most of the covariation between reading and arithmetic.

THE MODEL MAY BE CONCEPTUALIZED AS IN FIGURE I WHERE THE VARIABLES ARE AS FOLLOWS:

X_1	G	GRAPHIC MISCUES
X_2	P	PHONETIC MISCUES
X_3	S_M	SEMANTIC MISCUES
X_4	S_X	SYNTACTIC MISCUES
X_5	GP	GRAPHOPHONIC CUEING STRATEGY
X_6	SS	SYNTACTIC-SEMANTIC CUEING STRATEGY
X_7	Voc	VOCABULARY
X_8	LAN	LANGUAGE SKILLS
X_9	WORK	WORK-STUDY SKILLS
X_{10}	READ	READING COMPREHENSION
X_{11}	MATH	ARITHMETICAL COMPETENCY

FIGURE I HERE

THE RESULTS ARE PRESENTED IN DETAIL ELSEWHERE (BULCOCK, 1980). HERE WE COMPARE THE PARAMETER ESTIMATES FOR THE TWO ESTIMATING METHODS. IN FIGURE II THE 2SLS RESULTS ARE PRESENTED FOR THE FIRST STAGE OF THE MODEL.

FIGURE II HERE

OBVIOUSLY THE FIGURE II PATH COEFFICIENTS ARE DIFFICULT IF NOT IMPOSSIBLE TO INTERPRET.; SO ARE THE RESIDUALS. WHAT IS NEGATIVE R SQUARE? THE STRUCTURAL COEFFICIENTS IN FIGURE II ILLUSTRATE ALL THE PROBLEMS OF ACUTE MULTICOLLINEARITY.

(1) THE ESTIMATES ARE INFLATED.

(2) SEVERAL PATHS SEEM TO HAVE THE WRONG SIGN (COMPARE TABLE III).

(3) NONE OF THE PATH COEFFICIENTS ARE STATISTICALLY SIGNIFICANT INDICATING THAT THE STANDARD ERRORS ARE UNACCEPTABLY HIGH.

(4) THE RESIDUALS ARE UNINTERPRETABLE.

IN FIGURE III THE 2SNR RESULTS ARE PRESENTED FOR THE FIRST STAGE OF THE MODEL.

FIGURE III HERE

THE EVIDENCE STRONGLY SUGGESTS THAT THE PREFERENCE FOR AN ESTIMATOR WITH A SMALL BIAS AND SMALL VARIANCE TO ONE WITH NO BIAS BUT A LARGE VARIANCE IS ADVANTAGEOUS. IN OTHER WORDS, IF MULTICOLLINEARITY CAN BE REDUCED WITHOUT INTRODUCING TOO MUCH BIAS AS WAS ACCOMPLISHED BY THE 2SNR TECHNIQUE, THE LIMITATIONS OF THE STANDARD FEEDBACK ESTIMATING TECHNIQUE (2SLS) CAN BE OVERCOME.

THE ADVANTAGES OF 2SNR OVER 2SLS CAN BE NOTED AT THE SECOND STAGE OF MODEL ESTIMATION. SEE FIGURE IV.

FIGURE IV HERE

FIGURE I. CONCEPTUAL MODEL OF GRADE FOUR LITERACY AND NUMERACY

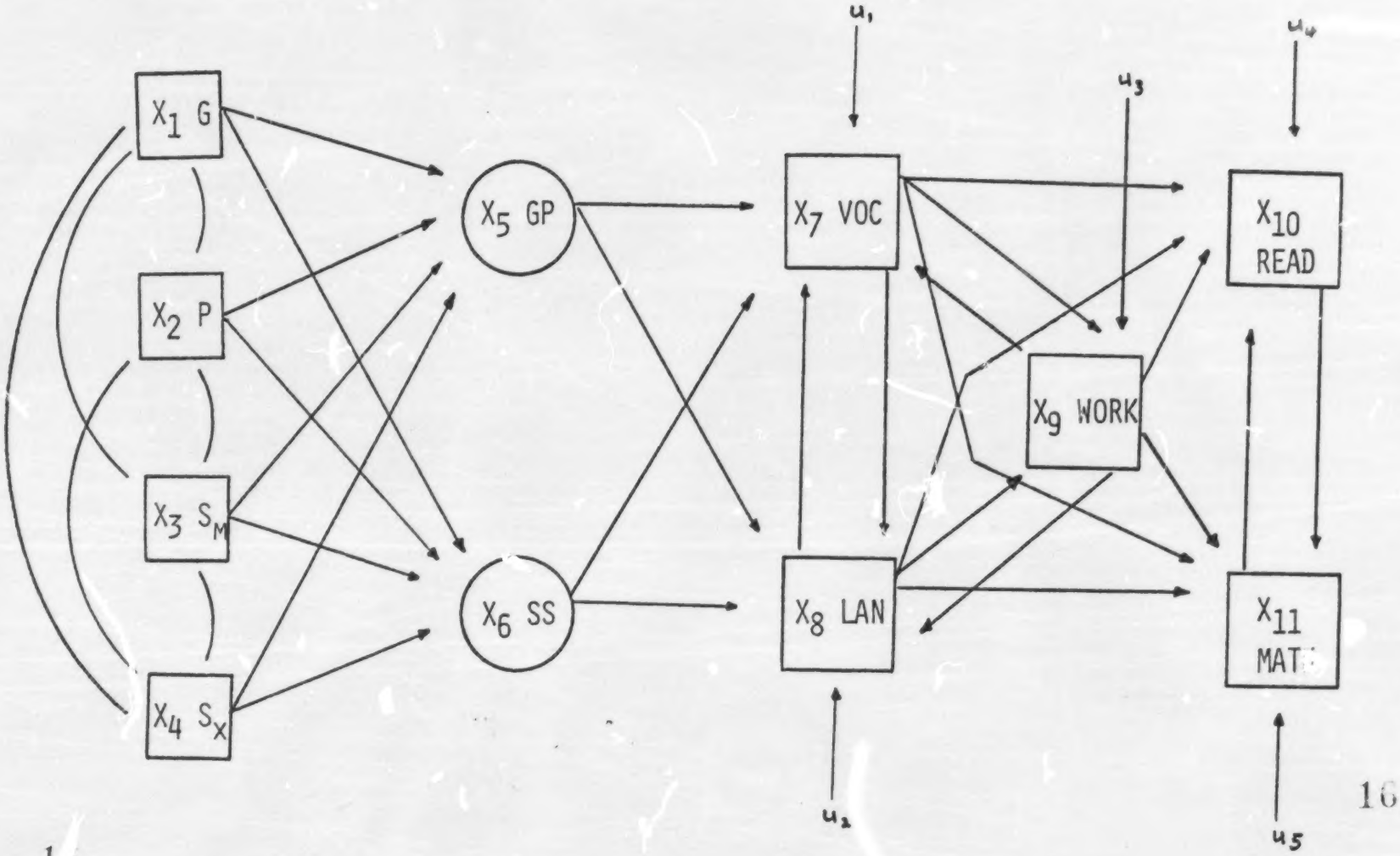


FIGURE II. 2SLS RESULTS FOR THE FIRST STAGE OF THE LITERACY-NUMERACY MODEL

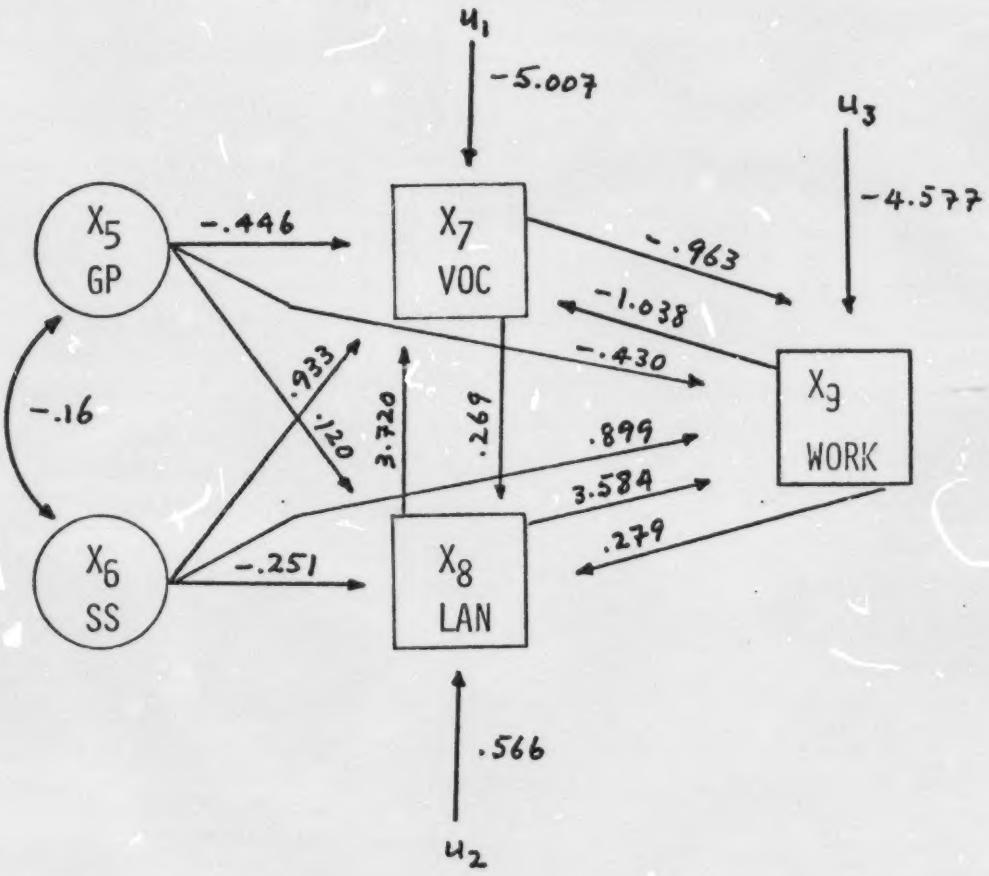


FIGURE III. 2SNR RESULTS FOR THE FIRST STAGE OF THE LITERACY-NUMERACY MODEL

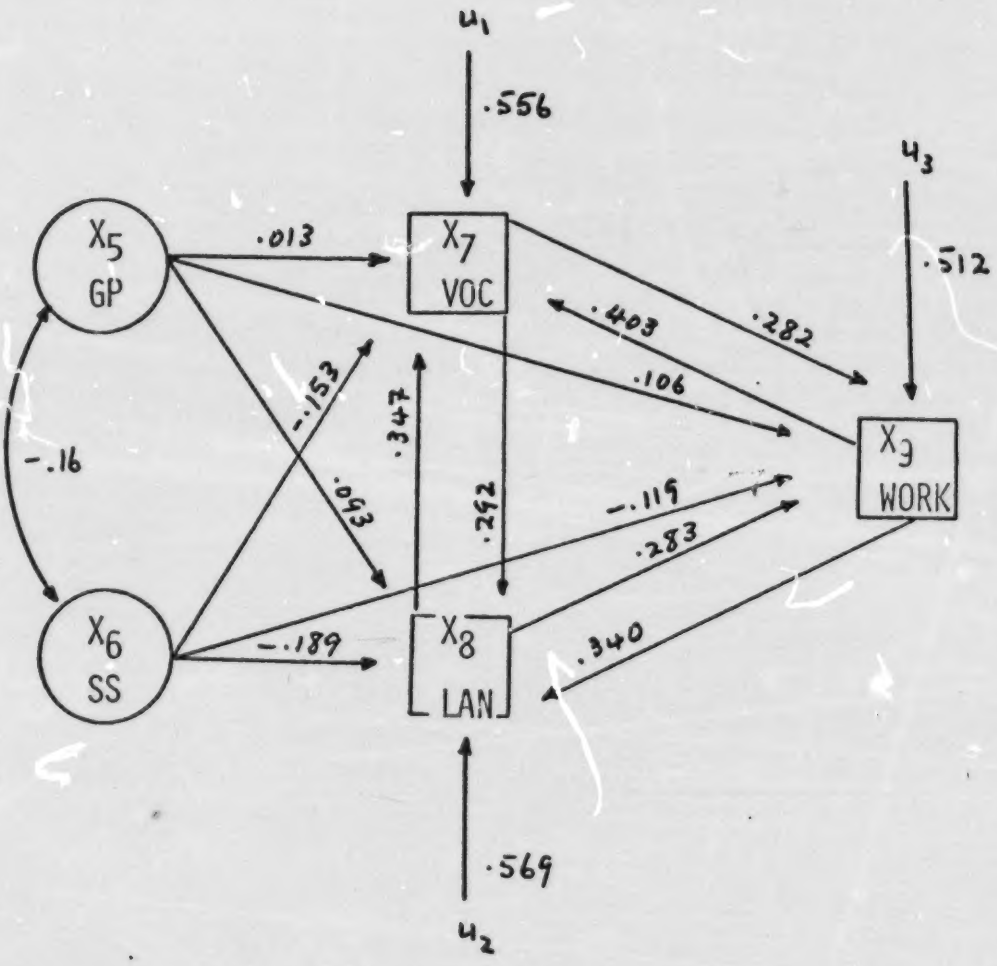
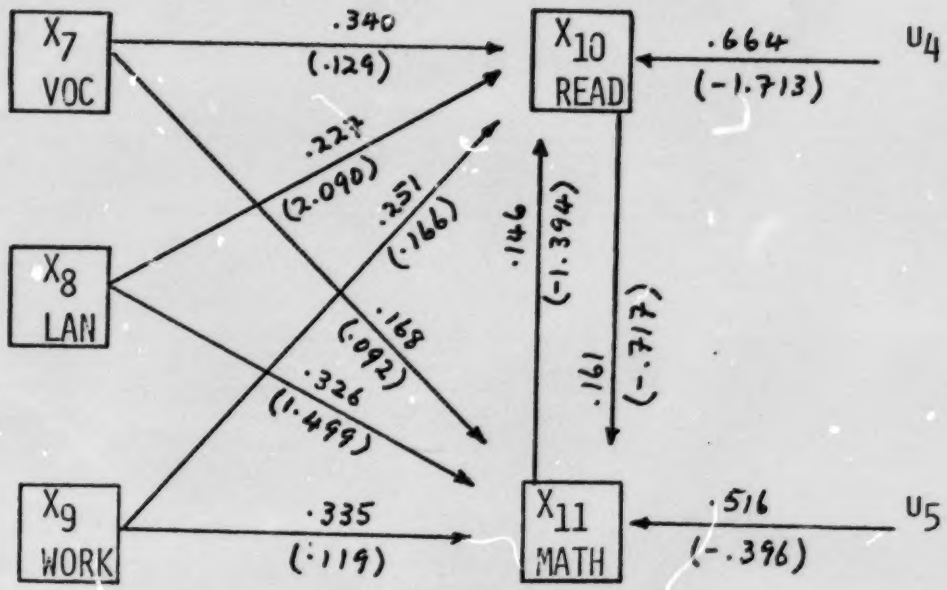


FIGURE IV. 2SNR AND 2SLS RESULTS FOR THE SECOND STAGE OF THE LITERACY-NUMERACY MODEL*



*2SNR COEFFICIENTS ARE ABOVE THE PATHS. 2SLS COEFFICIENTS ARE IN PARENTHESES BELOW THE PATHS.

VI. Concluding Remarks

(1) 2SLS feedback estimation is extremely sensitive to multicollinearity - a fact which negates several of the positive properties of the technique.

(2) The standard nonstatistical solutions to the problem include, improving the precision of measurement, improving the specification or design of the model, and increasing the sample size.

(3) The most important solution, however, is statistical; and in this regard the most promising technique seems to be the one developed by Hoerl and Kennard known as ridge regression.

(4) Ridge regression, however, is not problem free. Thus, though there are compelling reasons for using ridge regression as a collinearity reduction technique, it is nevertheless a stochastic procedure when it should be nonstochastic; and therefore does not yield a single unambiguous solution to the estimation problem, but rather a family of solutions.

(5) The question arises then whether a method might be devised which retains the advantages of ridge regression yet copes satisfactorily with its disadvantages. Bulcock and others (1980) have proposed use of a new estimating criterion

CALLED THE VARIANCE NORMALIZATION CRITERION AND HAVE SHOWN THAT THE CRITERION GOES MUCH OF THE DISTANCE TOWARDS SATISFYING THE QUESTION.

(6) THE NORMALIZATION RIDGE REGRESSION APPROACH WAS MODIFIED TO COPE WITH THE PROBLEM OF MULTIPLE FEEDBACK ESTIMATION IN CAUSAL MODELS. THE RESULTING TWO-STAGE NORMALIZATION REGRESSION APPROACH (2SNR) IS THE VARIANCE NORMALIZATION ANALOGUE TO TWO-STAGE LEAST SQUARES (2SLS).

(7) BECAUSE THE BETTER THE SPECIFICATION OF A FEEDBACK MODEL THE MORE ACUTE THE MULTICOLLINEARITY, THE NONRECURSIVE ESTIMATION OF ILL-CONDITIONED DATA PROVES IN PRACTICE TO BE FRAUGHT WITH DIFFICULTY.

(8) A NONRECURSIVE LITERACY-NUMERACY MODEL OF ELEMENTARY SCHOOLING WAS ESTIMATED BY BOTH 2SLS AND 2SNR METHODS. WHEN MULTICOLLINEARITY IS NEGLIGIBLE THE TWO METHODS GENERATE ALMOST IDENTICAL RESULTS. WHEN THE MULTICOLLINEARITY IS HIGH, HOWEVER, 2SLS RESULTS ARE UNINTERPRETABLE.

(9) IN CONTRAST, 2SNR RESULTS FOR THE LITERACY-NUMERACY MODEL WERE STABLE AND IN EVERY RESPECT CONGRUENT WITH THE THEORY. THUS, AT THE GRADE FOUR LEVEL INTUITIVE LINGUISTIC STRATEGIES WERE MORE INFLUENTIAL IN ACCOUNTING FOR LITERACY AND NUMERACY THAN FORMALLY LEARNED STRATEGIES; THE RELATIONSHIPS BETWEEN BASIC

SKILLS ON THE ONE HAND AND BETWEEN LITERACY AND NUMERACY ON THE OTHER WERE MUTUALLY REINFORCING, AND THE BASIC SKILLS WERE FOUND TO BE IMPORTANT VARIABLES MEDIATING THE RELATIONSHIP BETWEEN CUEING STRATEGIES AND THE ACQUISITION OF LITERACY AND NUMERACY.

(10) THOUGH THE 2SNR TECHNIQUE IN PRACTICE SEEMED TO HAVE IMPORTANT ADVANTAGES OVER 2SLS, MONTE CARLO SIMULATIONS HAVE TO BE CONDUCTED BEFORE IT CAN BE RECOMMENDED AS THE STANDARD NONRECURSIVE ESTIMATING PROCEDURE. NORMALIZATION REGRESSION RESEARCH IS STILL IN ITS INFANCY.